647510 **MSC1PhyC003x** **Seat No :________**

M.Sc.(Phy.) Semester - 1 (*CBCS*) Examination
Oct/Nov. -2019 - [NEW COURSE]
Quantum Mechanics I (Core (New))

Time: 2:30 Hours **Marks: 70**

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

Que.1 Answer any <u>seven</u> in brief: (14)

A. Prove that $[a, a^+] = 1$.

B. Prove that $[H, a^+]\ \hbar w a^+$.

C. What is zero point energy?

D. Why the positive sign in the exponent of the wave function is avoided in the solution of one dimensional quantum harmonic oscillator?

E. What is the main application of variation principle?

F. Using the first order time independent perturbation equation:
$(E_k - E_m)\, C_k^{(1)} + H'_{km} + W^{(1)}\, \delta_{km} = 0$
Obtain $C_k^{(1)}$ for $k \neq m$.

G. Why matrix is used in quantum mechanics as operators?

H. Define Dirac's Bra and Ket notations.

I. In the time dependent perturbation theory, $|C_m(t)|^2$ indicates what?

J. Why the WKB approximation is called as semi – classical apporoximation?

Que.2 Answer any <u>two</u> of the following:

(a) Solve the following equation for linear oscillator using power series method: (07)

$$\frac{d^2h}{d\xi^2} - 2\xi\,\frac{dh}{d\xi} + h(\in -1) = 0$$

Where, $\xi = \sqrt{\frac{mw}{\hbar}}\, x$ and $\epsilon = \frac{2E}{\hbar w}$

(b) Discuss the harmonic oscillator energy spectrum. (07)

(c) Define the operators a and a^+ and derive the oscillator Hamiltonian as (07)
$H = \hbar w\, (a^+ a + \frac{1}{2})$

Que.3 (a) Obtain the following expression for spherical polar coordinates: (07)

$$\overrightarrow{L^2} = -\hbar^2 \left[\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\,\frac{\partial}{\partial\theta}\right) + \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\phi^2}\right]$$

(b) How the orthogonal basis vectors $\overrightarrow{e_r}, \overrightarrow{e_\theta}$ *and* $\overrightarrow{e_\phi}$ in terms of the rectangular unit vectors $\overrightarrow{e_x}, \overrightarrow{e_y}$ *and* $\overrightarrow{e_z}$ and $\overrightarrow{e_x}, \overrightarrow{e_y}\ \overrightarrow{e_z}$ in terms $\overrightarrow{e_r}, \overrightarrow{e_\theta}$ *and* $\overrightarrow{e_\phi}$ can be expressed? Derive relations. (07)

OR

Que.3 (a) Using Schrodinger equation in spherical polar coordinates and applying the separable variable technique, separate radial part from the angular part and obtain following equation: (07)

$$\frac{1}{r^2}\,\frac{d}{dr}\left(r^2\,\frac{dR}{dr}\right) + \frac{2m}{\hbar^2}\left(E - V(r) - \frac{\gamma}{r^2}\right)R = 0$$

(b) For the attractive Coulomb potential of the nature $V(r) = \frac{-c}{r}$, where C = constant and C = Ze^2, solve the following radial equation. (07)

$$\frac{d^2 u_\gamma}{dr^2} + \left\{\frac{2m}{\hbar^2}\left\{E - \left(-\frac{c}{r}\right) - \frac{\varrho(\ell+1)}{r^2}\right\}\right\} \mu_e = 0$$ and obtain energy Eigen value $E_n = -\frac{mz^2e^4}{2\hbar^2 n^2}$

Que.4 Answer any two of following:

(a) Define time independent perturbation In the second order time independent theory, using the following expression (07)

$$(E_k - E_m)\Big(C_k^{(2)} + \textstyle\sum_n \left(H'_{kn} - W^{(1)}\delta_{kn}\right)\Big)C_n^{(1)}. - w^{(2)}\delta_{km} = 0$$

find out expression for $C_k^{(2)}$.

(b) In the time dependent perturbation theory for Fermi's golden rule, consider the following highly peaked function. (07)

$\mathrm{Sin}^2\, [(W_{mi} \neq W)\, t/2] \,/\, [(W_{mi} \neq W)/2]^2$

and apply the property of delta function and derive the following expression

$$W_{i\to m} = \frac{2\pi}{\hbar} |\langle \phi_m | \mathrm{H}_1 | \phi_i \rangle|^{\,2} \quad \varrho(\mathrm{E_m})|\mathrm{Em} = \mathrm{E}; \pm\hbar w$$

The bar over the expression indicates the average value over the final states.

(c) Obtain the solution of simple harmonic oscillator by variation method and prove that <H> = ½ $\hbar$ w, take trial wave function as $\underline{\Psi} = C_e^{-\lambda x^2}$ (07)

Que.5 Write short notes on any two:

(a) Spherical harmonics. (07)

(b) WKB approximation. (07)

(c) Variation method. (07)

(d) The raising and lowering operators. (07)

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